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Derived categories of graded gentle one-cycle algebras
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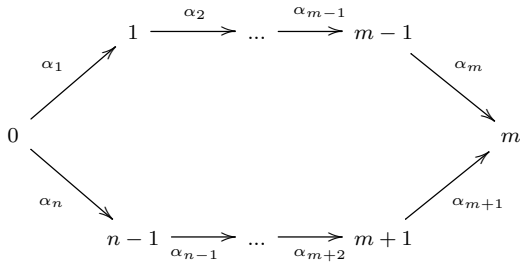
The representation type of finite-dimensional algebras is an important research theme in representation theory, which mainly studies the classification of objects in the derived category. Vossieck is undoubtedly a pioneer in this project. He introduced and classified derived discrete algebras in his celebrated paper [4]. The derived discrete algebras are defined to be ones whose derived category have only finitely many indecomposable objects with fixed cohomological dimension, which are classified into precisely two types in [4]: the algebras derived equivalent to hereditary algebras of finite type, and gentle one-cycle algebras that do not satisfy *clock condition*. The classification of finite-dimensional algebras into derived discrete ones and non-derived-discrete ones can be viewed as a version of second Brauer-Thrall type theorem for the derived category if we differentiate an object with its shifts. Another version of Brauer-Thrall type theorems for derived categories using cohomological range was obtained in [5].

Vossieck's work motivated the research of representation theory in the following two aspects. On one hand, the notation of discreteness was generalized for

general triangulated categories with respect to a bounded t-structure by Broomhead, Pauksztello Ploog in [3] and also with respect to bounded co-t-structure by Adachi, Mizuno, Yang in [1]. On the other hand, the gentle algebras are studied with algebraical method and also from geometrical origin. Both aspects provide the authors motivation to investigate the derived category of dg modules of graded gentle one-cycle algebras. The main results of this paper are as follows.

- **Investigate the relationship between various kinds of derived categories for arbitrary graded algebras.** This part provides us a diagram of all kinds of derived categories connected by forgetting functor and the totalize functor , which is an application of homological Koszul duality theory.
- **Classify the derived equivalent classes of graded gentle one-cycle algebras.** The classification is established using the dg version of Richard's theorem on derived equivalence, namely, by constructing a graded tilting complexes and then compute the endomorphism algebras. The description is divided into the cases of finite global dimension and infinite global dimension. To be more precise, let $\Gamma(n, m, r)$ be the quiver of the following form and $n > m > 0$ and $r \in \mathbb{Z}$,



Then for any graded one-cycle algebra A,

(1) if A is of finite global dimension, then A is equivalent to the graded path algebra $\Gamma(n, m, r)$ with an arrow of grading r;

(2) if A is of infinite global dimension, then A is equivalent to the quotient algebra of $\Gamma(n, n, r)$ with an arrow of grading r by the ideal generated by all paths of length two.

- **Describe the Auslander-Reiten quiver of finite-dimensional derived category $\mathcal{D}_{fd}(\Gamma(n, m, r))$.** The strategy of this part is to restrict the equivalence of derived categories as above to the equivalences on the level of perfect derived category and finite-dimensional derived category,

and connect the finite-dimensional derived category and the perfect derived category with Koszul duality. As applications, one can recover the result of Bobinski, Geiss, Skowronski in [2] for non-graded case, and one can also judge the derived equivalences between $\Gamma(n, m, r)$ directly by the triple (n, m, r).

Finally in this paper, the author propose an interesting conjecture involving the classification up to derived equivalences of graded gentle one-cycle algebras.

This paper is not only an important progress in the research of derived category of graded algebras, but also well-constructed for the convenience of readers. It also introduce some useful strategies, the Koszul duality for example, in the research of dg category in a complete and extensive manner. Both authors are experts in the theory of dg category, and anyone willing to learn about this subject is highly recommended to read this paper.

References

[1] T. Adachi, Y. Mizuno, D. Yang, *Discreteness of silting objects and t-structures of triangulated categories*, Proc. London Math. Soc. **118** (2019), 1-42.

[2] G. Bobinski, C. Geiss, and A. Skowronski, *Classification of discrete derived categories*, Cent. Eur. J. Math. **2** (2004), 19–49.

[3] N. Broomhead, D. Pauksztello, D. Ploog, *Discrete triangulated categories*, Bulletin of the London Mathematical Society, **50**(2018), 174-188.

[4] D. Vossieck, *The algebras with discrete derived category*, J. Algebra **243** (2001), 168-176.

[5] C. Zhang, Y. Han, *Brauer-Thrall type theorems for derived module categories*, Algebr Represent Theor **19**(2016)，1369–1386.