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Asymptotic behavior of random Fitzhugh-Nagumo systems driven by colored noise
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In this paper, the authors mainly consider three things. One is to prove the existence and uniqueness of random attractors for the FitzHugh-Nagumo system driven by colored noise with a nonlinear diffusion term in a bounded domain $\mathcal{O}$:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \triangle u + \alpha v = f(t,x,u) + g(t,x) + R(t,x,u)\zeta_\delta(\theta_t\omega), \\ \dfrac{\partial v}{\partial t} + \sigma v - \beta v = h(t,x) + \lambda\zeta_\delta(\theta_t\omega)v, \end{cases} \tag{1}$$

where α, β, σ and λ are positive constants, g and h are in $L^2_{loc}(\mathbb{R}, L^2(\mathcal{O}))$, which is a set of all locally square-integrable functions from $\mathbb{R}$ to $L^2(\mathcal{O})$, f and R are nonlinear functions which satisfy certain dissipative conditions and $\zeta_\delta(\theta_t\omega), \delta > 0$ is a colored noise which satisfies the linear stochastic differential equation

$$\begin{cases} d\zeta_\delta + \dfrac{1}{\delta}\zeta_\delta dt = \dfrac{1}{\delta}dW, \\ \zeta_\delta(\omega) = \dfrac{1}{\delta}\displaystyle\int_{-\infty}^0 e^{\frac{s}{\delta}} dW. \end{cases}$$

Here $W = W(t, \omega) = \omega(t)$ is a two-side real-value Wiener process on the classical Wiener probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and θ_t is the Wiener shift on it defined by $\theta_t \omega(\cdot) = \omega(t + \cdot) - \omega(t)$.

The second goal is to prove the existence of random attractors for Stratonovich stochastic differential system

$$\begin{cases} \dfrac{\partial u}{\partial t} - \triangle u + \alpha v = f(t, x, u) + g(t, x) + u \circ \dfrac{dW}{dt}, \\ \dfrac{\partial v}{\partial t} + \sigma v - \beta v = h(t, x) + \lambda v \circ \dfrac{dW}{dt}, \end{cases} \tag{2}$$

where $\circ$ means the Stratonovich differential.

The last objective is to prove the convergence of solutions and random attractors of system (1) with $R(t, x, u) = u$ as $\delta \to 0^+$.

This is a very interesting article. As is known to all, to study the pathwise dynamics of the stochastic FitzHugh-Nagumo system driven by a nonlinear multiplicative noise

$$\begin{cases} \dfrac{\partial u}{\partial t} - \triangle u + \alpha v = f(t, x, u) + g(t, x) + R(t, x, u) \circ \dfrac{dW}{dt}, \\ \dfrac{\partial v}{\partial t} + \sigma v - \beta v = h(t, x) + \lambda v \circ \dfrac{dW}{dt}, \end{cases} \tag{3}$$

one first needs to define a random dynamical system based on the solution operator of the equation. However, the existence of such a random dynamical system is unknown in general for a nonlinear function $R(t, x, u)$ in (3). So far, the pathwise dynamics such as random attractors and random invariant manifolds for equation (3) have been established only when $R(t, x, u)$ is either a linear function of u or independent of u. To avoid the difficulty, recently, many researchers introduce some stationary processes to replace white noise and study the pathwise dynamics of stochastic differential equation; see for instance [1, 2, 3, 4, 5].

In the present article, the authors consider the approximations of stochastic FitzHugh-Nagumo system (3) by a colored noise. Clearly, equation (1) driven by a colored noise is a random differential equation. As a consequence, its solutions generate a random dynamical system. Thus one can study its pathwise dynamical properties. Moreover, to prove that it is a good approximation, for the specific case $R(t, x, u) = u$, the authors prove the approximate relationships of solutions and attractors of equation (1) and equation (2). This provides a method to study the pathwise dynamics of stochastic equation (3).

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