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Holomorphic Campanato type spaces over Carleson tubes and Bergman metric balls

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The paper under review gives a series of characterizations of the holomorphic Campanato type spaces in terms of Carleson tubes and Bergman metric balls in the unit ball.

It is shown in the paper that, for a holomorphic function f in the unit ball of the complex n-dimensional Euclidean space $\mathbb{C}^n$, if $0 < \eta < 2$ and $p \geq 1$, then the following conditions are equivalent:

(1)

$$\sup_{\zeta\in\mathbb{S}_n, r>0} \frac{1}{(v_\alpha(B(\zeta,r)))^\eta} \int_{B(\zeta,r)} |f(z) - f_{\alpha,B(\zeta,r)}|^p \mathrm{d}v_\alpha(z) < \infty,$$

where $B(\zeta,r) = \{z \in \mathbb{B}_n : |1 - \langle z,\zeta\rangle| < r\}$ be the Carleson tube at ζ with radius r, and

$$f_{\alpha,B(\zeta,r)} = \frac{1}{v_\alpha(B(\zeta,r))} \int_{B(\zeta,r)} f(z)\mathrm{d}v_\alpha(z).$$

(2) For each $r > 0$ and $\zeta \in \mathbb{S}_n$ there is a number c such that

$$\frac{1}{(v_\alpha(B(\zeta,r)))^\eta} \int_{B(\zeta,r)} |f(z) - c|^p \mathrm{d}v_\alpha(z) \lesssim 1.$$

(3)

$$\sup_{a\in\mathbb{B}_n}\left((1-|a|^2)^{(1-\eta)(n+1+\alpha)}\int_{\mathbb{B}_n}|f(\varphi_a(z))-f(a)|^p\mathrm{d}v_\alpha(z)\right)^{\frac{1}{p}}<\infty,$$

where $\varphi_a(z)$ is the involutive automorphism of $\mathbb{B}_n$.

(4)

$$\sup_{a\in\mathbb{B}_n}\left(\int_{\mathbb{B}_n}|\widetilde{\nabla}f(z)|^p\frac{(1-|a|^2)^{(2-\eta)(n+1+\alpha)}}{|1-\langle z,a\rangle|^{2(n+1+\alpha)}}\,\mathrm{d}v_\alpha(z)\right)^{\frac{1}{p}}<\infty,$$

where $\widetilde{\nabla}f(z)$ is the invariant gradient of f at z.

Moreover,

(5) If $p>2(\eta-1)(n+1+\alpha)$, then the above conditions are equivalent to

$$\sup_{a\in\mathbb{B}_n}\left(\int_{\mathbb{B}_n}\frac{|Rf(z)|^p(1-|\varphi_a(z)|^2)^{\eta(n+1+\alpha)}}{(1-|z|^2)^{\eta(n+1+\alpha)-p-\alpha}}\,\mathrm{d}v(z)\right)^{\frac{1}{p}}<\infty,$$

where $Rf(z)$ is the radial derivative of f at z.

(6) If $0<\eta<1$, then the above conditions are equivalent to

$$\sup_{\zeta\in\mathbb{S}_n,r>0}\left(\frac{1}{(v_\alpha(B(\zeta,r)))^\eta}\int_{B(\zeta,r)}|f(z)|^p\mathrm{d}v_\alpha(z)\right)^{\frac{1}{p}}<\infty.$$

It is also proved that if $r>0$, $\alpha>-1$ $\frac{n}{n+1+\alpha}<\eta<2$ and $p\geq 1$, f is holomorphic, then the following conditions are equivalent:

(i)

$$\sup_{\zeta\in\mathbb{S}_n,r>0}\frac{1}{(v_\alpha(D(a,r)))^\eta}\int_{D(a,r)}|f(z)-f_{\alpha,D(a,r)}|^p\mathrm{d}v_\alpha(z)<\infty,$$

where $D(a,r)$ be the Bergman metric ball centered at ζ with radius r.

(ii)

$$\sup_{a\in\mathbb{B}_n}\frac{1}{(v_\alpha(D(a,r)))^\eta}\int_{D(a,r)}|f(z)-f(a)|^p\mathrm{d}v_\alpha(z)<\infty.$$

(iii)

$$\sup_{a\in\mathbb{B}_n}\frac{1}{(v_\alpha(D(a,r)))^\eta}\int_{D(a,r)}|f(z)-f_{\alpha,D(a,r)}|^p\mathrm{d}v_\alpha(z)<\infty.$$

(iv) For every $z\in\mathbb{B}_n$ there is a complex number c_z such that

$$\frac{1}{(v_\alpha(D(z,r)))^\eta}\int_{D(z,r)}|f(w)-c_z|^p\mathrm{d}v_\alpha(w)\lesssim 1.$$

These characterizations reveal that for some parameters, these Campanato type spaces are the same as the well-studied $F(p,q,s)$ spaces; see Ref.[7] therein. This actually provides some new characterizations to those $F(p,q,s)$ spaces.