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n-Abelian quotient categories
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Cluster-tilting theory provides a framework for the categorification of Fomin-Zelevinsky's cluster algebras. It investigates cluster-tilting objects and the representation theory of their endomorphism algebras, which is used to construct abelian quotient categories from triangulated categories by factoring out any cluster tilting subcategory. Let n be a positive integer. With the development of higher cluster-tilting theory and higher Auslander-Reiten theory, Geiss-Keller-Oppermann [1] introduced the notion of $(n+2)$-angulated categories, which are "higher dimensional" analogues of triangulated categories. Similarly, Jasso [2] introduced the notion of n-abelian categories, which are "higher dimensional" analogues of abelian categories. It is natural to study whether we can obtain n-abelian quotients from $(n+2)$-angulated categories by factoring out any cluster tilting subcategory. Jacobsen-Jørgensen [3] made a first attempt. With the help of the definition of cluster-tilting subcategories in $(n+2)$-angulated categories due to Oppermann-Thomas [6], they considered $(n+2)$-angulated categories with Serre functors and gave a positive answer. The paper under review is devoted to introducing the general case of the n-abelian quotients.

Let $\mathcal{C}$ be an $(n+2)$-angulated category with an n-suspension functor Σ^n and $\mathcal{X}$ be a cluster-tilting subcategory of $\mathcal{C}$. The authors first show that the quotient category

$\mathcal{C}/\mathcal{X}$ is an n-abelian category. Then they describe the projective objects and injective objects in $\mathcal{C}/\mathcal{X}$. By proving that $\mathcal{C}/\mathcal{X}$ is projective generated, they obtain that $\mathcal{C}/\mathcal{X}$ is an n-cluster-tilting subcategory of the abelian category mod$\Sigma^{-n}\mathcal{X}$. In particular, if $\mathcal{C}$ has a Serre functor, they prove that mod$\Sigma^{-n}\mathcal{X}$ is Gorenstein of Gorenstein dimension at most n. These generalize recent results of Jacobsen-Jørgensen [3] and Koenig-Zhu [4].

Based on the work of Jacobsen-Jørgensen [3], the main results of the paper under review are reasonable. But the proofs are interesting. The authors provide different and elegant proofs by using the method of right $(n+2)$-angulated categories and the fact that a projective generated n-abelian category is equivalent to an n-cluster tilting subcategory of an abelian category with enough projectives [5].

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