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MODIFIED ADOMIAN DECOMPOSITION METHOD FOR SOLVING FUZZY INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. In this paper, Adomian decomposition method and modified Adomian decomposition method are applied to find the approximate solutions of fuzzy Volterra integro-differential equations. The reliability of the method and reduction in the size of the computation give these methods a wider applicability. Also, the behavior of the solution can be formally determined via the analytical approximation. Finally, an example is included to demonstrate the validity and applicability of the proposed methods.

1. Introduction

The fuzzy integral equation plays a vital role with in many disciplines of sciences, engineering and mathematics. By use of fuzzy integral and fuzzy integro-differential equations with exact parameters within many modeling physical problems is not quite easy or better to say impossible in real problems. To overcome this difficulty one of the most recent approach is to use fuzzy concept. Basic concept of fuzzy was first introduced by professor Zadeh in 1965 after his publication on fuzzy set theory [1, 2]. Solving fuzzy integro-differential equations requires appropriate and applicable definitions of fuzzy function, the fuzzy derivative and the fuzzy integral of a fuzzy function. The fuzzy differential and integral equations are important part of the fuzzy analysis theory and they have the important value of theory and application in control theory. Later, the area of interest in FIEs has been expanded into the fuzzy integro-differential equations (FIDEs). The FIDEs take the form of both FDEs and FIEs [5]. A particular class of FIDEs is known as fuzzy Volterra integro-differential equations. The existence and uniqueness of FIDEs and FVIDEs solutions were investigated by Park and Jeong in [19], Hajighasemi et al. in [7], and Zeinali et al. in [20]. Mikaeilvand

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et al. in [18] presented the numerical examples of FVIDEs using the differential transform method. In [1, 2], Allahviranloo et al. proposed a new technique to solve the FVIDEs using definition of generalized differentiability. Hamoud and Ghadle in [13] presented the homotopy analysis method for the first order fuzzy Volterra-Fredholm integro-differential equations.

Motivated by the above work, in this paper we discuss some methods for fuzzy integro-differential equations of the form:

$$\tilde{u}'(t) = \tilde{f}(t, \tilde{u}(t)) + \int_a^t \tilde{K}(t,s)\tilde{u}(s)ds, \ t \in (a, b], \tag{1.1}$$

with the initial condition

$$\tilde{u}(a) = \tilde{u}_0, \tag{1.2}$$

where function $\tilde{f} : \mathbb{R} \times \mathbb{R}_F \longrightarrow \mathbb{R}_F$, and $\tilde{K}(t, s)$ are continuous and $\tilde{u}_0$ is a fuzzy number.

2. Fuzzy Integro-Differential Equations

When a physical system is modeled under the differential sense, it finally gives a fuzzy integral equation or a fuzzy integro-differential equation and hence, the solution of fuzzy integro-differential equations have a major role in the fields of science and engineering. Nonlinear fuzzy integro-differential equations are usually hard to solve analytically and exact solutions are scarce. Therefore, they have been of great interest by several authors.

Consider the first order fuzzy integro-differential equations of the form:

$$\tilde{u}'(t) = \tilde{f}(t, \tilde{u}(t)) + \int_a^t \tilde{K}(t,s)\tilde{u}(s)ds, \ t \in (a, b], \tag{2.1}$$

with the initial condition

$$\tilde{u}(a) = \tilde{u}_0, \tag{2.2}$$

The fuzzy integro-differential equations (2.1) is equivalent to the following system

$$\underline{u}'(t,r) = \underline{f}(t, \underline{u}(t,r)) + \int_a^t \underline{K(t,s)u(s,r)}ds, \tag{2.3}$$

$$\overline{u}'(t,r) = \overline{f}(t, \overline{u}(t,r)) + \int_a^t \overline{K(t,s)u(s,r)}ds, \tag{2.4}$$

we write the parametric form of the given equation (2.1) as follows

$$\underline{u}(t,r) = \underline{u}_0 + \int_a^t \underline{f}(s, \underline{u}(s,r))ds + \int_a^t \int_a^s \underline{K(t,s)u(s,r)}drds, \tag{2.5}$$

$$\overline{u}(t,r) = \overline{u}_0 + \int_a^t \overline{f}(s, \overline{u}(s,r))ds + \int_a^t \int_a^s \overline{K(r,s)u(s,r)}drds \tag{2.6}$$

where $[\tilde{u}_0]_r = [\underline{u}(t_0; r), \overline{u}(t_0; r)], \ r \in [0, 1]$.

$$\underline{K(t,s)u(s,r)} = \begin{cases} K(t,s)\underline{u}(s,r), & K(t,s) \geq 0, \\ K(t,s)\overline{u}(s,r), & K(t,s) < 0. \end{cases}$$

$$\overline{K(t,s)u(s,r)} = \begin{cases} K(t,s)\overline{u}(s,r), & K(t,s) \geq 0, \\ K(t,s)\underline{u}(s,r), & K(t,s) < 0. \end{cases}$$

3. Basic Concepts

The concept of fuzzy numbers is generalized classical real numbers and we can say that a fuzzy number is a fuzzy subset of the real line which has some extra properties. The concept of fuzzy number is vital for fuzzy analysis, fuzzy integral equations and fuzzy differential equations, and a very helpful tool in different applications of fuzzy sets. Basic definition of fuzzy numbers are given in [1,8]

Definition 3.1. [1] A fuzzy number is a fuzzy set like $u\colon \mathbb{R} \to [0,1]$ with the following properties:

- u is upper semi-continuous function.
- u is fuzzy convex, i.e, $u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for all $x, y \in \mathbb{R}, \lambda \in [0,1]$.
- u is normal, i.e, $\exists x_0 \in \mathbb{R}$ for which $u(x_0) = 1$.
- $\sup u = \{x \in \mathbb{R} | u(x) > 0\}$ is the support of the u, and its closure $\mathrm{cl}(\sup u)$ is compact.

Let $\mathbb{E}$ be the set of all fuzzy numbers on $\mathbb{R}$. The $(\alpha - cut)$ α-level set of a fuzzy number $u \in \mathbb{E}, 0 \leq \alpha \leq 1$, denoted by $[u]_\alpha$, is defined as

$$[u]_\alpha = \begin{cases} \{x \in \mathbb{R} : u(x) \geq \alpha\}, & 0 < \alpha \leq 1, \\ cl(\sup u), & \alpha = 0. \end{cases}$$

where $cl(\sup u = x \in \mathbb{R} | u(x) > 0)$ indicates the closure of the support of u. It is clear that the α-level set of a fuzzy number is a closed and bounded interval $[\underline{u}(\alpha), \overline{u}(\alpha)]$, where $\underline{u}(\alpha)$ indicates the left-hand end point of $[u]_\alpha$ and $\overline{u}(\alpha)$ indicates the right-hand end point of $[u]_\alpha$. Since every $u \in \mathbb{R}$ can be regarded as a fuzzy number $\tilde{u}$ defined by

$$\tilde{u}(t) = \begin{cases} 1, & t = u \\ 0, & t \neq u. \end{cases}$$

An equivalent parametric definition is also given in [8].

Definition 3.2. [8] A fuzzy number $\tilde{u}$ in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(\alpha)$, $\overline{u}(\alpha), 0 \leq \alpha \leq 1$, which satisfy the following requirements:

- $\underline{u}(\alpha)$ is a bounded non-decreasing left continuous function in $(0,1]$, and right continuous at 0.
- $\overline{u}(\alpha)$ is a bounded non-increasing left continuous function in $(0,1]$, and right continuous at 0.
- $\underline{u}(\alpha) \leq \overline{u}(\alpha), \quad 0 \leq \alpha \leq 1$.

A crisp number α is simply represented by $\underline{u}(\alpha) = \overline{u}(\alpha) = \alpha, 0 \leq \alpha \leq 1$.

We recall that for $a < b < c$ which $a, b, c \in \mathbb{R}$, the triangular fuzzy number $u = (a, b, c)$ determined by a, b, c are given such that $\underline{u}(\alpha) = a + (b-a)\alpha$ and $\overline{u}(\alpha) = c - (c-b)\alpha$ are the end points of the α-level sets, for all $\alpha \in [0,1]$.

The Hausdorff distance between fuzzy numbers given by $d : \mathbb{E} \times \mathbb{E} \to \mathbb{R}_+ \cup \{0\}$.

$$d(u, \upsilon) = \sup_{\alpha \in [0,1]} \max\{|\underline{u}(\alpha) - \underline{\upsilon}(\alpha)|, |\overline{u}(\alpha) - \overline{\upsilon}(\alpha)|\},$$

where $u = (\underline{u}(\alpha), \overline{u}(\alpha))$, $\upsilon = (\underline{\upsilon}(\alpha), \overline{\upsilon}(\alpha)) \subset \mathbb{R}$ is utilized in [?]. And it is easy to see that d is a metric in $\mathbb{E}$ and has the following properties:

- $d(u + \rho, \upsilon + \rho) = d(u, \upsilon), \quad \forall\ u, \upsilon, \rho \in \mathbb{E}.$
- $d(ku, kv) = |k| d(u, \upsilon), \quad \forall\ k \in \mathbb{R}\ u, v \in E.$
- $d(\omega + \upsilon, \rho + e) \leq d(\omega, \rho) + d(\upsilon, e), \quad \forall\ \omega, \upsilon, \rho, e \in \mathbb{E}.$
- $(d, \mathbb{E})$ is a complete metric space.

Definition 3.3. [1] Let $f : \mathbb{R} \to \mathbb{E}$ be a fuzzy valued function. If for arbitrary fixed $t_0 \in R$ and $\forall\ \epsilon > 0$, $\exists\ \delta > 0$ such that $|t - t_0| < \delta \Longrightarrow |f(t) - f(t_0)| < \epsilon$, then f is said to be continuous.

Theorem 3.4. *[8] Let $f(x)$ be a fuzzy-valued function on $[a, \infty)$ and it is represented by $(\underline{f}(x, \alpha), \overline{f}(x, \alpha))$. For any fixed $t \in [0, 1]$ assume $\underline{f}(x, \alpha)$ and $\overline{f}(x, \alpha)$ are Riemann-integrable on $[a, b]$ for every $b \geq a$, and assume there are two positive $\underline{M}(\alpha)$ and $\overline{M}(\alpha)$ such that $\int_a^b \left|\underline{f}(x, \alpha)\right| dx \leq \underline{M}(\alpha)$ and $\int_a^b \left|\overline{f}(x, \alpha)\right| dx \leq \overline{M}(\alpha)$ for every $b \geq a$. Then $f(x)$ is improper fuzzy Riemann-integrable on $[a, \infty)$ and the improper fuzzy Riemann-integral is a fuzzy number. Furthermore, we have*

$$\int_a^\infty f(x)dx = \Big(\int_a^\infty \underline{f}(x, \alpha)dx, \int_a^\infty \overline{f}(x, \alpha)dx \Big).$$

Proposition 3.5. *[8]. If each of $f(x)$ and $g(x)$ is fuzzy-valued function and fuzzy Riemman integrable on $\Omega = [a, \infty)$ then $f(x) + g(x)$ is fuzzy Riemman integrable on Ω. Moreover, we have*

$$\int_\Omega (f(x) + g(x))dx = \int_\Omega f(x)dx + \int_\Omega g(x)dx.$$

Definition 3.6. [8] The integral of a fuzzy function was define by using the Riemann integral concept. Let $f : [a, b] \to \mathbb{E}$, for each partition $P = \{t_0, t_1, ..., t_n\}$ of $[a, b]$ and for arbitrary $\xi_i \in [t_{i-1}, t_i], 1 \leq i \leq n$, suppose

$$R_p = \sum_{i=1}^n f(\xi_i)(t_i - t_{i-1}).$$

$$\Delta := \max |t_i - t_{i-1}|, 1 \leq i \leq n.$$

The definite integral of $f(t)$ over $[a, b]$ is

$$\int_a^b f(t)dt = \lim_{\Delta \to 0} R_p$$

provided that this limit exists in the metric d. If the fuzzy function $f(t)$ is continuous in the metric d, it is definite integral exists, and also

$$\underline{\int_a^b f(t, r)dt} = \int_a^b \underline{f}(t, r)dt,$$

$$\overline{\int_a^b f(t,r)dt} = \int_a^b \overline{f}(t,r)dt$$

It should be noted that the fuzzy integral can be additionally defined using the Lebesgue-type approach [8]. Nonetheless, if f is continuous, the two approaches yield the similar value. More details about the properties of the fuzzy integral are given in [8].

4. Description of the Methods

We will talk briefly on some dependable techniques for fathoming this sort of equations, where subtleties can be found in [3, 4, 6, 9–12, 14, 17].

4.1. **Adomian Decomposition Method (ADM).** First of all, we discuss Adomian decomposition method [15,16]. Consider the following fuzzy integro-differential equations of the form

$$\underline{u}(t,r) = \underline{u}_0 + \int_a^t \underline{f}(s,\underline{u}(s,r))ds + \int_a^t \int_a^s \underline{K(t,s)u(s,r)}drds, \tag{4.1}$$

$$\overline{u}(t,r) = \overline{u}_0 + \int_a^t \overline{f}(s,\overline{u}(s,r))ds + \int_a^t \int_a^s \overline{K(r,s)u(s,r)}drds. \tag{4.2}$$

The ADM assume an infinite series solution for the unknowns functions $[\underline{u},\overline{u}]$, given by

$$\underline{u}(x) = \sum_{i=0}^{\infty} \underline{u}_i(x),$$

$$\overline{u}(x) = \sum_{i=0}^{\infty} \overline{u}_i(x). \tag{4.3}$$

The $\tilde{A}_n = [\underline{A}_n, \overline{A}_n], n \geq 0$, are the so-called Adomian polynomial, and $g(t,r) = u_0 + \int_a^t f(s,u(s))ds$ we get

$$\underline{u}_0 = \underline{g}(t,r),$$

$$\underline{u}_1 = \int_a^t \underline{K(t,s)A_0}\ ds,$$

$$\cdots$$

$$\underline{u}_{n+1} = \int_a^t \underline{K(t,s)A_n}\ ds,\ \ n > 1, \tag{4.4}$$

and

$$\overline{u}_0 = \overline{g}(t,r),$$

$$\overline{u}_1 = \int_a^t \overline{K(t,s)A_0}ds,$$

$$\cdots$$

$$\overline{u}_{n+1} = \int_a^t \overline{K(t,s)A_n}\ ds,\ \ n > 1. \tag{4.5}$$

We approximate $\tilde{u}(t,r) = [\underline{u}(t,r), \overline{u}(t,r)]$ by

$$\underline{\varphi}_n = \sum_{i=0}^{n-1} \underline{u}_i(t,r),$$
$$\overline{\varphi}_n = \sum_{i=0}^{n-1} \overline{u}_i(t,r),$$

where

$$\lim_{n\to\infty} \underline{\varphi}_n = \underline{u}(t,r), \ \lim_{n\to\infty} \overline{\varphi}_n = \overline{u}(t,r).$$

4.2. **Modified Adomian Decomposition Method (MADM).** In this part, the extended MADM [11] is used to find approximate of nonlinear integro-differential equation (2.1). This method is based on the assumption that the function $g(t,r)$ can be divided into two parts, namely $g_1(t,r)$ and $g_2(t,r)$. Under this assumption we set

$$\underline{u}(t,r) = \underline{g}_1(t,r) + \underline{g}_2(t,r) + \int_a^t \int_a^s \underline{K(t,s)u(s,r)} drds, \tag{4.6}$$

$$\overline{u}(t,r) = \overline{g}_1(t,r) + \overline{g}_2(t,r) + \int_a^t \int_a^s \overline{K(r,s)u(s,r)} drds, \tag{4.7}$$

Hence, we can write the following iteration formula:

$$\begin{aligned}
&\underline{u}_0 = \underline{g}_1(t,r)\\
&\underline{u}_1 = \underline{g}_2(t,r) + \int_a^t \underline{K(t,s)A_0}\ ds\\
&\dots\\
&\underline{u}_{n+1} = \int_a^t \underline{K(t,s)A_n}\ ds, \ n > 1,
\end{aligned} \tag{4.8}$$

and

$$\begin{aligned}
&\overline{u}_0 = \overline{g}_1(t,r)\\
&\overline{u}_1 = \overline{g}_2(t,r) + \int_a^t \overline{K(t,s)A_0} ds\\
&\dots\\
&\overline{u}_{n+1} = \int_a^t \overline{K(t,s)A_n}\ ds, \ n > 1.
\end{aligned} \tag{4.9}$$

We approximate $\tilde{u}(t,r) = [\underline{u}(t,r), \overline{u}(t,r)]$ by

$$\underline{\varphi}_n = \sum_{i=0}^{n-1} \underline{u}_i(t,r),$$
$$\overline{\varphi}_n = \sum_{i=0}^{n-1} \overline{u}_i(t,r),$$

where

$$\lim_{n\to\infty} \underline{\varphi}_n = \underline{u}(t,r), \ \lim_{n\to\infty} \overline{\varphi}_n = \overline{u}(t,r).$$

5. Numerical Results

In this section, The Comparison is made between ADM, and MADM for solving fuzzy Volterra integro-differential equations. The efficiency of method is shown in terms of error which is estimated by $\underline{E}(t;r) = |\underline{u}_E(t;r) - \underline{u}_A(t;r)|$ and $\overline{E}(t;r) = |\overline{u}_E(t;r) - \overline{u}_A(t;r)|$.

Example 1. Consider the following fuzzy Volterra integro-differential equation

$$u'(t;r) = C\frac{1}{12t}(36 - 5t^4) + \int_0^t (t^2 + s^2)u(s;r)ds, \tag{5.1}$$

$$C = [(r^5 + 2r)t^3, (6 - 3r^3)t^3], \ u(0) = [0,0], \ 0 \le s \le t \le 1.$$

The equivalent system is

$$\underline{u}'(t;r) = \frac{rt^2}{12}(r^4 + 2)(36 - 5t^4) + \int_0^t (t^2 + s^2)\underline{u}(s;r)ds, \ \underline{u}(0;r) = 0,$$
$$\overline{u}'(t;r) = \frac{t^2}{12}(6 - 3r^3)(36 - 5t^4) + \int_0^t (t^2 + s^2)\overline{u}(s;r)ds, \ \overline{u}(0;r) = 0,$$

The exact solutions are

$$\underline{u}(t;r) = (r^5 + 2r)t^3, \ \overline{u}(t;r) = (6 - 3r^3)t^3.$$

Table 1. Left Bound of Error $\underline{E}(t = 0.5, r)$.

r	$\underline{E}_{ADM}$	$\underline{E}_{MADM}$
0.0	0.00000000	0.0000000
0.1	7.956810E-11	4.823713E-10
0.2	7.992855E-10	9.654662E-10
0.3	5.998988E-10	1.452902E-9
0.4	5.033500E-10	1.954086E-9
0.5	4.133730E-10	2.487103E-9
0.6	4.360880E-10	3.081621E-9
0.7	4.806100E-10	3.781770E-9
0.8	4.596280E-10	4.649055E-9
0.9	2.190025E-9	5.765231E-9
1.0	2.493445E-9	7.235207E-9

Table 2. Right Bound of Error $\overline{E}(t = 0.5, r)$.

r	$\overline{E}_{ADM}$	$\overline{E}_{MADM}$
0.0	3.986892E-9	1.447042E-8
0.1	3.985401E-9	1.446318E-8
0.2	3.974945E-9	1.441254E-8
0.3	3.946571E-9	1.427506E-8
0.4	2.891310E-9	1.400736E-8
0.5	2.800214E-9	1.356601E-8
0.6	2.664307E-9	1.290761E-8
0.7	2.474641E-9	1.198874E-8
0.8	1.222250E-9	1.076599E-8
0.9	1.898170E-9	1.195954E-8
1.0	1.493445E-9	1.235207E-8

6. Conclusions

The modified Adomian decomposition method is successfully applied to find the approximate solution of fuzzy Volterra integro-differential equation. The reliability of the method and reduction in the size of the computational work give this method a wider applicability, the obtained numerical results are compared with the exact solutions, for problem (5.1) at t = 0.5. For problem (5.1), in Tables 1 and 2, it is observed that the errors at t = 0.5 obtained by the ADM is competitive with the MADM. The method is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of linear and nonlinear integro-differential equations.

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